The initial impetus for this development came from a specific interest in a small antenna in the form of a disc-loaded monopole with no dielectric substrate [5]. From this, the author perceived the opportunity for the more general formula reported herein.

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Frequency Dependent Characteristics of Microstrips on Anisotropic Substrates

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***Abstract*—Frequency dependent characteristics are discussed for the microstrip line on anisotropic substrate (original line) from the extension of the results obtained in an isotropic substrate case. In approximating the original line by the equivalent line on an isotropic substrate, it is best to maintain h and w of the equivalent line equal to those of the original line. The reason is that the inflection frequency f_i is a function of w and h and that f_i plays an important role in calculating the dispersion. Three approximate dispersion formulas are derived owing to this idea. The results obtained by these formulas are compared with the other available theoretical and experimental results for sapphire substrates. Good agreement is seen.**

I. Introduction

Many articles have appeared giving design data for a microstrip which is an essential part of the integrated circuit in modern microwave devices. Sapphire has several advantages as a substrate material, so that its higher cost is offset [1]. Sapphire is a uniaxial crystal; it is therefore anisotropic. Being anisotropic, the microstrip lines possess characteristics which differs somewhat from that of lines on isotropic substrates.

The characteristics of microstrip lines on anisotropic substrates have been already investigated with a high degree of accuracy in the quasistatic approximation (see [9]–[11]). On the other hand, many available analyses for the case of isotropic substrate also can be used for the case of anisotropic substrate by using the transformation from anisotropic to isotropic problems [2], [4], [5], [10].

However, recent developments require the operation of a microstrip line at higher frequencies. Some authors have theoretically studied the frequency dependent characteristics for the case of anisotropic substrates [4], [5] and have checked the validity of the equivalent line of the isotropic substrate for such a case [4], [5]. However, there was a large discrepancy between the effective dielectric constants of the two corresponding lines at the higher frequency. The experimental investigation and the empirical formula have been compared to a sapphire substrate [3].

On the other hand, the computer-aided design of microstrip circuits requires accurate and reliable information on the dispersive behavior. For the case of an isotropic substrate, a few approximate formulas satisfying the CAD requirement have been derived (see [6], [12]). The author has also formulated a new approximate dispersion formula, and discussed the important role of the inflection frequency f_i on the dispersion curves, and the influence of the structural parameters, ϵ^*(relative dielectric constant), h(substrate thickness), and w/h(shape ratio) to the dispersion curves [12].

In this short paper, the simple approximate dispersion formulas are derived for a microstrip line on an anisotropic substrate. As an example, the numerical results of a case of sapphire substrate have been compared with the other available theoretical results [4], [5] and experimental data [3].

II. Approximate Dispersion Formula

Consider the microstrip line of width w on an anisotropic substrate of thickness h whose permittivity tensor (in two-dimensional space) is presented by

$$\bar{\bar{\epsilon}} = \begin{pmatrix} \epsilon_\xi^* & 0 \\ 0 & \epsilon_\eta^* \end{pmatrix} \epsilon_0 \tag{1}$$

where ϵ_ξ^* denotes the relative dielectric constant in the direction of the ξ-axis, ϵ_η^* denotes the relative dielectric constant in the direction of the η-axis, and ϵ_0 is the permittivity of free-space. The structure is shown in Fig. 1. The $\xi-\eta$ coordinates are identical to the principal axes of this substrate and can be obtained by rotating the $x-y$ coordinates with the angle γ. In designing such a line, the characteristic impedance Z and the phase velocity v(wave length λ, propagation constant β) must be obtained. Let us express the effective dielectric constant at a frequency f by $\epsilon_{\mathrm{eff},f}^*$. These values above can be obtained as follows [7], [8], [12]:

$$Z = \frac{\eta_0 D}{\frac{C_0}{\epsilon_0}\sqrt{\epsilon_{\mathrm{eff},f}^*}} \tag{2}$$

$$\frac{v}{v_0} = \frac{\lambda}{\lambda_0} = \frac{\beta_0}{\beta} = \frac{1}{\sqrt{\epsilon_{\mathrm{eff},f}^*}} \tag{3}$$

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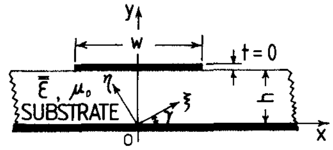

Fig. 1. Structure of microstrip line on anisotropic substrate.

where $\eta_0(=\sqrt{\mu_0/\epsilon_0})$ denotes the impedance of free-space, C_0 denotes the line capacitance for $\epsilon_x^*=\epsilon_y^*=1$, v_0 is the speed of light in free space, $\lambda_0(=v_0/f)$ is the wavelength in free space, and β_0 $(=2\pi f/v_0)$ is the propagation constant in free space. Also, $D=1$ in [7] and $D=(\epsilon_{\text{eff},f}^*-1)/(\epsilon_{\text{eff},0}^*-1)$ in [8]. The C_0/ϵ_0 has already been tabulated with a very high degree of accuracy for a wide range of w/h [9], [11]. Equations (2) and (3) contain explicitly $1/\sqrt{\epsilon_{\text{eff},f}^*}$. It is better to express an approximate dispersion formula as the form of $1/\sqrt{\epsilon_{\text{eff},f}^*}$.

The author presented the dispersion properties for the case of an isotropic substrate in [12]. The order of a magnitude of $\epsilon_{\text{eff},f}^*$ is determined by the effective dielectric constant $\epsilon_{\text{eff},0}^*$ at zero frequency, that is, determined by w/h and ϵ^*. Since the dispersion curve is related to the inflection frequency, the value of f_t is very important in deriving the approximate formula. The author believes that the properties obtained for the isotropic substrate case are also valid for the case of anisotropic substrates. We already know [12] that good design data can be obtained from both Yamashita's formula (formula Y) [6] and a modification to Schneider's formula (formula K) [12] for the case of isotropic substrates.

We consider the application of both formulas to the anisotropic substrate case. However, it needs the skillful idea since the substrate is anisotropic. The equivalent line of width w on an isotropic substrate is represented by equivalent substrate thickness and relative dielectric constant of $\alpha h/\langle(\alpha^2-1)\cos^2\gamma+1\rangle(\alpha=\sqrt{\epsilon_\eta^*/\epsilon_\xi^*})$ and $\sqrt{\epsilon_\xi^*\epsilon_\eta^*}$, respectively, as shown in [2], [4], [5], [10], and [11], and are highly accurate for the line capacitance C at zero frequency. To approximate the dispersive behavior of the line on an anisotropic substrate (original line) by that in this equivalent line will lead to a large discrepancy at the higher frequency, because the f_t is the function of not only a relative dielectric constant and a shape ratio, but the substrate thickness [12], and the shape ratio and the substrate thickness in the equivalent line are different from those in the original line. The author believes that it is better to maintain the h and w of an original structure. Therefore, we must consider the equivalent relative dielectric constant $\epsilon_{\text{eq},f}^*$ of the substrate as a function of frequency f in order to make the effective dielectric constant of the equivalent line equal to that of the original line $\epsilon_{\text{eff},f}^*$.

Next, we consider the approximate dispersion formula owing to the idea mentioned above. We can obtain, with the high degree of accuracy, the line capacitance of the original line at zero frequency by the Green's function technique [9], [10] or by approximate formulas [10], [11]. Therefore, we can get the effective and equivalent relative dielectric constants, $\epsilon_{\text{eff},0}^*$ and $\epsilon_{\text{eq},0}^*$, respectively.

The high frequency limit of the effective dielectric constant $\epsilon_{\text{eff},\infty}^*$ is always the relative dielectric constant in the direction perpendicular to the ground conductor irrespective of the line dimensions. This is supported by the facts shown in Section V of [5] for the case which the structure axes are identical to the principal axes of substrate, that is, $\gamma=0$, in Fig. 1.

Schneider's formula (formula K) for isotropic case [12] can be modified to be used for an anisotropic case as follows:

$$\frac{1}{\sqrt{\epsilon_{\text{eff},f}^*}}=\frac{\dfrac{1}{\sqrt{\epsilon_{\text{per}}^*}}\left(\dfrac{f}{f_K}\right)^2+\dfrac{1}{\sqrt{\epsilon_{\text{eff},0}^*}}}{\left(\dfrac{f}{f_K}\right)^2+1} \tag{4}$$

where

$$f_K=\frac{v_0\tan^{-1}\left(\epsilon_{\text{eq},0}^*\sqrt{\dfrac{\epsilon_{\text{eff},0}^*-1}{\epsilon_{\text{eq},0}^*-\epsilon_{\text{eff},0}^*}}\right)}{2\pi h\left(1+\dfrac{w}{h}\right)\sqrt{\epsilon_{\text{eq},0}^*-\epsilon_{\text{eff},0}^*}} \tag{5}$$

$\epsilon_{\text{eq},0}^*$ is the equivalent relative dielectric constant at zero frequency for an isotropic substrate on which the microstrip line (w and h being identical to those of the original line) has the same effective dielectric constant $\epsilon_{\text{eff},0}^*$ as the latter line at zero frequency. ϵ_{per}^* is the relative dielectric constant in the direction perpendicular to the ground conductor. For the substrate shown in Fig. 1,

$$\epsilon_{\text{per}}^*=\frac{1}{m\left(\epsilon_\xi^*,\epsilon_\eta^*,\dfrac{\pi}{2}-\gamma\right)} \tag{6}$$

where

$$m(\epsilon_\xi^*,\epsilon_\eta^*,\theta_\xi)=\sqrt{\frac{1}{\epsilon_\xi^*}\cos^2\theta_\xi+\frac{1}{\epsilon_\eta^*}\sin^2\theta_\xi} \tag{7}$$

and m is the metric factor in the radial direction with the angle θ_ξ from the one of principal axes, the ξ-axis [10].

Yamashita's formula (formula Y) [6] can be modified similarly as follows:

$$\frac{1}{\sqrt{\epsilon_{\text{eff},f}^*}}=\frac{\left(\dfrac{f}{f_Y}\right)^{1.5}+4}{\left(\dfrac{f}{f_Y}\right)^{1.5}\sqrt{\epsilon_{\text{per}}^*}+4\sqrt{\epsilon_{\text{eff},0}^*}} \tag{8}$$

where

$$f_Y=\frac{v_0}{4\pi h\sqrt{\epsilon_{\text{eq},0}^*-1}\left[0.5+\left\{1+2\log\left(1+\dfrac{w}{h}\right)\right\}^2\right]}. \tag{9}$$

The author proposes the following formula (formula YK) by using formula K and formula Y:

$$\frac{1}{\sqrt{\epsilon_{\text{eff},f}^*}}=\frac{1}{2}[U1\times\text{eq.}(4)+U2\times\text{eq.}(8)] \tag{10}$$

where $U1=1$ or 0 and $U2=1$ or 2 for the frequency f satisfying $(4)\geqslant(8)$ or $(4)<(8)$, respectively.

III. Dispersion Behavior in a Sapphire Substrate Case

As an example, a sapphire substrate case ($\epsilon_\parallel^*=11.6$, $\epsilon_\perp^*=9.4$) is calculated. The C_0/ϵ_0, C/ϵ_0, and $\epsilon_{\text{eq},0}^*$ must be obtained before the three formulas are used. Table I shows these values calculated by the Green's function technique [9]. The high degree of accuracy of this technique has already been discussed in previous papers [9], [11]. The C_0/ϵ_0 in Table I are calculated by conformal mapping for $0.02\leqslant w/h\leqslant 10$ and by the Green's function technique for $20<w/h\leqslant 100$. Table II shows the comparison of the present results with other results [3]. A very good agreement is seen.

TABLE I
THE C_0/ϵ_0, C/ϵ_0, AND $\epsilon^*_{eq,0}$ FOR THE MICROSTRIP LINE ON SAPPHIRE SUBSTRATE

w/h	C_0/ε_0	$\varepsilon^*_x=9.4$ $\varepsilon^*_y=11.6$ C/ε_0	$\varepsilon^*_x=9.4$ $\varepsilon^*_y=11.6$ $\varepsilon^*_{eq,0}$	$\varepsilon^*_x=11.6$ $\varepsilon^*_y=9.4$ C/ε_0	$\varepsilon^*_x=11.6$ $\varepsilon^*_y=9.4$ $\varepsilon^*_{eq,0}$
0.02	1.04869	6.5466	10.67	6.3051	10.24
0.04	1.18587	7.4922	10.69	7.1777	10.20
0.1	1.43375	9.2596	10.75	8.7842	10.15
0.2	1.70270	11.265	10.81	10.571	10.10
0.4	2.09392	14.352	10.91	13.256	10.01
0.7	2.56364	18.309	11.01	16.608	9.92
1	2.97989	22.016	11.09	19.688	9.85
2	4.23155	33.963	11.26	29.462	9.70
4	6.52694	57.513	11.40	48.603	9.58
7	9.79678	92.622	11.48	77.104	9.51
10	12.9813	127.62	11.51	105.50	9.48
20	23.362	244.04	11.56	199.91	9.44
40	43.766	476.47	11.58	388.34	9.42
70	74.103	824.83	11.59	670.70	9.41
100	104.32	1173.0	11.59	952.93	9.41

TABLE II
COMPARISON OF THE $\epsilon^*_{eq,0}$ AND $\epsilon^*_{eff,0}$ FOR THE MICROSTRIP LINE ON SAPPHIRE SUBSTRATE

line No.		1	2	3	4	5	6	7	8	9	10	11
w (mm)		4.490	2.445	1.921	1.068	0.642	0.415	0.275	0.178	0.111	0.073	0.050
h (mm)		0.491	0.492	0.512	0.496	0.510	0.507	0.502	0.480	0.507	0.502	0.496
$\varepsilon^*_{eq,0}$	K	11.50	11.43	11.39	11.28	11.15	11.04	10.96	10.90	10.82	10.78	10.74
	E	11.50	11.43	11.39	11.29	11.18	11.08	10.99	10.91	10.82	10.76	10.73
$\varepsilon^*_{eff,0}$	K	9.740	9.065	8.736	8.105	7.577	7.242	7.002	6.822	6.641	6.536	6.460
	E	9.733	9.057	8.732	8.108	7.594	7.263	7.019	6.832	6.640	6.524	6.448
C_0/ε_0	K	12.078	7.5980	6.2497	4.4140	3.3180	2.7323	2.3353	2.0428	1.7453	1.5675	1.4363
K = Green's function technique [9], [10] F = Edwards and Owens [3]												

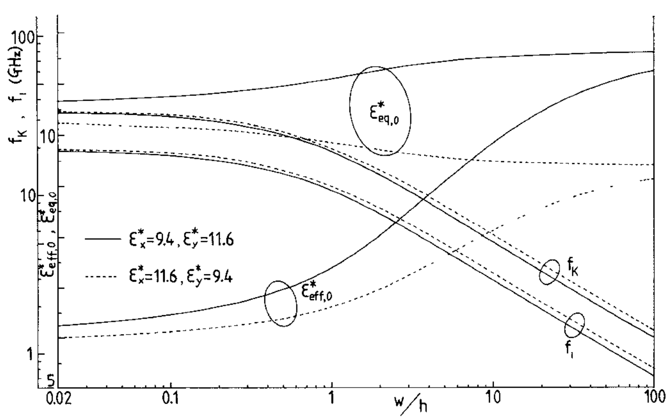

Fig. 2. The $\epsilon^*_{eff,0}, \epsilon^*_{eq,0}, f_K, f_i$ for the microstrip line on sapphire substrate The f_K and f_i are calculated as $h = 1$ mm. The f_K and f_i for other $h = x$ mm can be obtained by dividing the values read in this figure by x.

Fig. 2 shows $\epsilon^*_{eff,0}$, $\epsilon^*_{eq,0}$, f_K, and f_i versus the w/h. Both $\epsilon^*_{eq,0}$ and $\epsilon^*_{eff,0}$ approach the values of $\sqrt{\epsilon^*_{\parallel} \epsilon^*_{\perp}}$ (= about 10.442) and $\sqrt{\epsilon^*_{\parallel} \epsilon^*_{\perp}}/2$ (= about 5.221), respectively, when $w/h \to 0$. On the other hand, the $\epsilon^*_{eq,0}$ and $\epsilon^*_{eff,0}$ approach the values of the relative dielectric constant in the direction perpendicular to the ground conductor when $w/h \to \infty$. The f_i is the inflection frequency at which the second-order derivative of $1/\sqrt{\epsilon^*_{eff,f}}$ in (4) with respect of f is zero. The f_i is given by

$$f_i = \frac{1}{\sqrt{3}} f_K. \tag{11}$$

The f_i has an important role on the dispersion curves because the $1/\sqrt{\epsilon^*_{eff,f}}$ at the f_i is given, for any ϵ^*_ξ, ϵ^*_η, and γ, by

$$\frac{1}{\sqrt{\epsilon^*_{eff,f_i}}} = \frac{\dfrac{3}{\sqrt{\epsilon^*_{eff,0}}} + \dfrac{1}{\sqrt{\epsilon^*_{per}}}}{4}. \tag{12}$$

The $1/\sqrt{\epsilon^*_{eff,f}}$ variates from the $1/\sqrt{\epsilon^*_{eff,0}}$ at zero frequency and to the $1/\sqrt{\epsilon^*_{per}}$ at infinite frequency. In the curves, f_i occurs when $1/\sqrt{\epsilon^*_{eff,f}}$ is approximately the value in (12) (see Fig. 3). Therefore, we can draw the approximate curves of $1/\sqrt{\epsilon^*_{eff,f}}$ versus f if the $1/\sqrt{\epsilon^*_{eff,0}}$, $1/\sqrt{\epsilon^*_{per}}$, and f_i are given. Fig. 3 compares the results of formula K(dot–dash–line), formula Y(broken line), and formula YK(solid line) with the other available results [3]–[5]. The dotted line denotes the theoretical results; [4] for $w/h = 0.1$, 1, 10, and [5] for the other two dotted lines. The experimental results [3] are marked by an "x", in Fig. 3. We see that formula K gives the upper bound and formula Y the lower bound in the frequency range less than about the f_i or f_K, and that formula YK

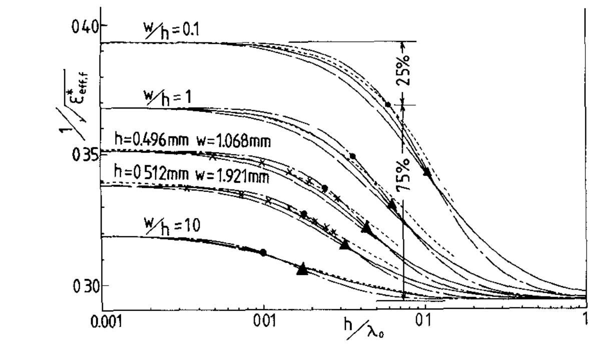

Fig. 3. The $1/\sqrt{\epsilon^*_{\mathrm{eff},f}}$ for the microstrip line on sapphire substrate. $\epsilon^*_\xi = 9.4$, $\epsilon^*_\eta = 11.6$, $\gamma = 0$ in Fig 1 —·— formula $K(4)$, — — formula $Y(8)$, —— formula $YK(10)$, ------ theoretical results; [4] for $w/h = 0.1$, 1, 10, and [5] for the other two dotted lines "x" experimental results [3], ● $f_i(11)$, ▲ $f_K(5)$

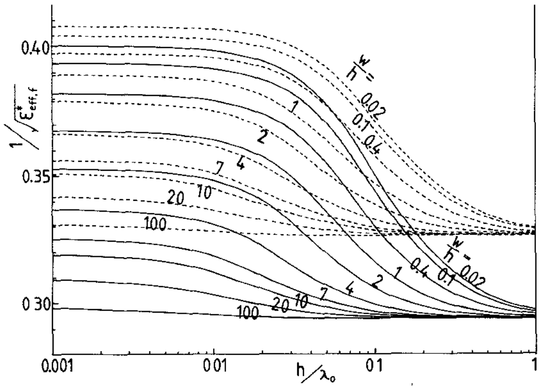

Fig. 4. The $1/\sqrt{\epsilon^*_{\mathrm{eff},f}}$ for the microstrip line on sapphire substrate calculated by formula $YK(10)$. —— $\epsilon^*_x = 9.4$, $\epsilon^*_y = 11.6$; ------ $\epsilon^*_x = 11.6$, $\epsilon^*_y = 9.4$.

shows good agreement with the theoretical and experimental results and provides good design data. Fig. 4 shows the numerical results of formula YK. The equations also show the influence of the cutting angle of substrate on the dispersion properties.

IV. Conclusion

Three simple approximate dispersion formulas (K, Y, YK) have been derived. The results obtained have been compared with other available results with good agreement. We have found that formula YK gives good design data.

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Important Role of Inflection Frequency in the Dispersive Property of Microstrip Lines

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***Abstract*—It is represented that the inflection frequency f_i has the important role in the dispersive property of microstrip lines. This f_i is related to the coupling between the TEM mode and the TM_0 mode. Using f_i, an approximate dispersion formula is derived by improving Schneider's formula. The results obtained by the present formula are compared with the other available results; good agreement is seen.**

A microstrip transmission line is an essential part of an integrated circuit which is a fundamental component in modern microwave devices. With its increasing use at higher frequencies, a number of workers have theoretically studied the dispersive properties of microstrip lines [1]–[5] (good bibliographies are given in [6] and [10]). However, their analyses usually require a complicated computer program and, in some cases, enormous computing time. Recently, their results have been compared [6]. On the other hand, the computer-aided design of microstrip circuits requires accurate and reliable information on the dispersive behavior. A few approximate equations satisfying these requirements have been formulated [8]–[15], [17].

In a microstrip geometry, transverse TM- and TE-wave modes exist. Only even-order TM surface-wave modes and odd-order TE surface-wave modes are possible [18]. The TM_0 mode is dominant since it has a zero frequency cutoff [18], while higher order

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